Section 4.1: Sequences

1. Note: In Problem 15, the sequence should be $0, \ -\frac{1}{2}, \frac{2}{3}, \ -\frac{3}{4}, \frac{4}{5}, \ \dots$

2. **Sequence** - an ordered set of elements
   Term, initial term, final term, infinite sequence, general formula

3. Example #1: List the first several terms of the following sequences.
   (a) $b_j = \frac{5-j}{5+j}, \quad \forall\, j \in \mathbb{Z}, j \geq 1$
   (b) $a_n = \left\lfloor \frac{n}{2} \right\rfloor \cdot 2, \quad \forall\, n \in \mathbb{Z}, n \geq 1$
   (c) $f_0 = f_1 = 1,\ f_n = f_{n-1} + f_{n-2}, \forall\, n \geq 2$ (Fibonnaci sequence)

4. Example #2: Find a general (explicit) formula for the n^{th} term of the sequence.
   (a) $1, -4, 9, -16, 25, \dots$
   (b) $1 - \frac{1}{2}, \frac{1}{2} - \frac{1}{3}, \frac{1}{3} - \frac{1}{4}, \frac{1}{4} - \frac{1}{5}, \dots$
   (c) $3, 6, 12, 24, 48, 96, \dots$

5. Example #3: Summation and Product Notation (Compute the following)
   (a) $\sum_{m=0}^{3} \frac{1}{2^m}$
   (b) $\prod_{j=0}^{4} (-1)^j$
   (c) $\sum_{k=1}^{n} \frac{1}{k(k+1)}$ (Use Partial Fractions)
   (d) $\sum_{k=1}^{30} f_n - f_{n-1}$ (Fibonnaci sequence)

6. Factorials: $n!$ and Binomial Coefficients (Pascal's Triangle)
   (a) $\frac{8!}{3!4!}$
   (b) $\frac{(n+1)}{n!}$
   (c) $\frac{n!}{(n-4)!}$

7. $\lim_{n \to \infty} \frac{f_n}{f_{n-1}} = \frac{1+\sqrt{5}}{2} = \varphi(n)$ (Golden Ratio)

- Since the Renaissance, artists and architects have proportioned works to approximate $\varphi(n)$
- The golden rectangle (ratio of longer side to shorter is golden ratio): proportion is believed to be aesthetically pleasing

8. Construction of a golden rectangle:

   (a) Construct a unit square.

   (b) Draw a line from the midpoint of one side to an opposite corner.

   (c) Use that line as the radius to draw an arc that defines the long dimension of the rectangle.

   (d) When the square is removed, the remaining rectangle has the same proportion. (Golden Spiral)

9. Change of Variable:

   (a) $\sum_{j=0}^{20} \frac{1}{j+1}$ (Substitute $k = j + 1$)

   (b) $\sum_{i=5}^{12} \frac{i}{i-1}$ (Substitute $k = i - 1$)

   (c) $\sum_{k=1}^{n+1} \left(\frac{k}{n+k}\right)$ (Substitute $j = n + k$)