Section 4.2: Mathematical Induction I

1. Examples: Falling dominoes and Marietta College book brigade

2. Example: For all integers $n \geq 8$, n cents can be obtained using 3¢ and 5¢ coins.
   Formally: For all integers $n \geq 8$, $P(n)$ is true, where $P(n)$ is the sentence
   "n cents can be obtained using 3¢ and 5¢ coins."
   Look at how to obtain $(k+1)$¢ from k¢ if previous did/didn't include a 5¢ coin.

3. **Principle of Mathematical Induction:** (Validity of method is taken as an axiom.)
   Let $P(n)$ be a property that is defined for integers n, and let a be a fixed integer. Suppose the following two statements are true:

   (a) $P(a)$ is true.

   (b) For all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true.

   Then, $P(n)$ is true for all integers $n \geq a$.

4. History of Mathematical Induction:

   - First used by Italian scientist Francesco Maurolico in 1575.
   - Pierre de Fermat ("method of infinite descent") and Blaise Pascal used it in 17$^{\text{th}}$ century.
   - In 1883, Augustus DeMorgan described the process and called it *mathematical induction*.

5. Equivalent Statement of Mathematical Induction:
   If S is a set of integers satisfying $a \in S$ and $k \in S \Rightarrow k+1 \in S$, then $k \in S$, $\forall\ k \in \mathbb{Z}$, $k \geq a$.

6. Proving by Mathematical Induction: To prove "For all integers $n \geq a$, $P(n)$ is true."

   (a) Show that the property is true for $n = a$.

   (b) Show that for all integers $k \geq a$, $P(k)$ is true $\Rightarrow$ $P(k+1)$ is true.
   To do this,

   - Assume that the property is true for $n = k$, where $k \in \mathbb{Z}$, $k \geq a$.
   - Show that the property is true for $n = k+1$.

7. Proposition 4.2.1
   For all integers $n \geq 8$, $P(n)$ is true, where $P(n)$ is the sentence
   "n cents can be obtained using 3¢ and 5¢ coins."
   Prove using mathematical induction.

8. Examples: Prove the following using mathematical induction.

   (a) Theorem 4.2.2:
   $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ for all integers $n \geq 1$. (Also prove without induction.)

   (b) Theorem 4.2.3: (Sum of a Geometric Sequence)
   $\forall\ r \in \mathbb{R}$, $r \neq 1$ and any non-negative integer n,

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}.$$

(c) (p. 226, pr. 13)

$$\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}, \ \forall\, n \in \mathbb{Z}, n \geq 2.$$