Section 5.1: Basics of Sets

1. Set Theory (Georg Cantor, late 19^{th} century): foundation of mathematical thought.
   All mathematical objects can be defined in terms of sets.
2. **Set** and **elements** are undefined terms: a set is a collection of elements.
3. **Axiom of Existence:** a set is completely determined by its elements (order is irrelevant).
4. Notations:
   (a) {Golf, Tennis, Soccer}: List all element (in braces): repeats, ordering irrelevant
   (b) $\{\text{Golf}\} \neq \text{Golf}$
   (c) Sets can be elements of other sets: $\{2, \{2\}\}$ (Power set)
   (d) Set builder notation: $\{x \in \mathbb{R} : 1 < x \leq 4\}$
5. Subsets:
   - $A \subset B \Leftrightarrow \ \forall\, x$, if $x \in A$ then $x \in B$.
   - A is not a subset of B (notation) $\Leftrightarrow\ \exists\, x$ s.t. $x \in A$ and $x \notin B$.
   - Proper subset. (Notations)
6. Venn Diagrams: With and without boxes; Drawing $A = B$ and $A \subseteq B$.
7. Notation: Distinguish between $\in$ and $\subseteq$: $2 \in \{1, 2\}$, $\{2\} \subseteq \{1, 2\}$.
8. Set Equality: $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.
9. Example #1:
   Let $A = \{2q - 7 : q \in \mathbb{Z}\}$
   Let $B =$ the set of all odd integers.
   Show that $A = B$.
10. Definitions:
    Universal set, union, intersection, difference (relative complement) $B - A$, complement
11. Examples: Go through previous definitions with examples and Venn Diagrams
12. Define empty set (null set), look at in terms of intersections
13. A and B are disjoint $\Leftrightarrow A \cap B = \varnothing$.
14. $A_1, A_2, \ldots, A_n$ are mutually disjoint $\Leftrightarrow A_i \cap A_j = \varnothing$ if $i \neq j$.
15. Partition: $\{A_1, A_2, \ldots, A_n\}$ is a partition of A if $A = A_1 \cup A_2 \cup \cdots \cup A_n$ and the A_i's are mutually disjoint.
16. Example: Look at partitions of $\mathbb{Z}$ (mod n)
17. Power Set ($\mathcal{P}(A)$)
18. Ordered n-tuples (pair, triple, etc.): When are they equal?
19. Cartesian Product: $A \times B$