

Section 7.1: Functions

1. Definitions:

- function $f : X \rightarrow Y$
- domain, codomain
- image, range
- inverse image (preimage)

2. Functions defined by arrow diagrams (give examples of functions and nonfunctions)

3. Functions defined by formulas (squaring function, constant function, etc.)

4. Notation: This book uses f to refer to function, $f(x)$ to refer to value of f at x .

5. Suppose $f : X \rightarrow Y$ and $g : X \rightarrow Y$. Then $f = g$ iff

$$f(x) = g(x) \forall x \in X.$$

6. Example #1: If $f(x) = x + 2$ and $g(x) = \frac{x^2 + 3x + 2}{x + 1}$, does $f = g$?

7. Example #2: (p. 400, pr. 9)

Let F and G be functions from \mathbb{R} to \mathbb{R} . Define $F \cdot G : \mathbb{R} \rightarrow \mathbb{R}$ and $G \cdot F : \mathbb{R} \rightarrow \mathbb{R}$ by

$$(F \cdot G)(x) = F(x) \cdot G(x) \text{ for all } x \in \mathbb{R},$$

$$(G \cdot F)(x) = G(x) \cdot F(x) \text{ for all } x \in \mathbb{R}.$$

Does $F \cdot G = G \cdot F$?

8. Sequence: A function $f : \mathbb{Z} \rightarrow \mathbb{R}$.

9. Example #3: (p. 400, pr. 12)

Find functions defined on $\mathbb{Z} \cup \{0\}$ that define the sequences whose first six terms are:

(a) $1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}$

(b) $0, -2, 4, -6, 8, -10$

10. Checking whether a function is well defined: Let $f : \mathbb{Q} \rightarrow \mathbb{Z}$ be defined by

$$f\left(\frac{m}{n}\right) = m, \quad \forall m, n \in \mathbb{Z}, n \neq 0.$$

Is f well defined?

11. Example #4 (p. 400, pr. 16) Log functions.

Find the exact values for each of the following quantities.

(a) $\log_3 81$

(b) $\log_2 1024$

(c) $\log_3 \frac{1}{27}$

(d) $\log_2 1$

(e) $\log_{10} \frac{1}{10}$

(f) $\log_3 3$

(g) $\log_2(2^k)$

12. Example #4 (p. 400, pr. 23)

Let $A = \{2, 3, 5\}$ and $B = \{x, y\}$. Let p_1 and p_2 be the projections of $A \times B$ onto the first and second coordinates, i.e. $p_1(a, b) = a$ and $p_2(a, b) = b$.

(a) Find $p_1(2, y)$ and $p_1(5, x)$. What is the range of p_1 ?

(b) Find $p_2(2, y)$ and $p_2(5, x)$. What is the range of p_2 ?

13. A Boolean function f is a function whose domain is the set of all ordered n -tuples of 0's and 1's and whose co-domain is the set $\{0, 1\}$. ($f : \{0, 1\}^n \rightarrow \{0, 1\}$)