Section 1.2 - Analytic Technique: Separations of Variables

1. Standard form for a first-order differential equation:

$$\frac{dy}{dt} = f(t, y)$$

(Recall independent variable (t), dependent variable (function))
A **solution** of the differential equation is a function of the independent variable.

2. Solutions to a differential equation:
$P(t) = e^{kt}$, $P(t) = ce^{kt}$, and $P(t) = 0$ are solutions to the differential equation

$$\frac{dP}{dt} = kP,$$

but $P(t) = (1/2)kt^2$ is not.

3. Example: Solve the initial value problem

$$\frac{dy}{dt} = \cos t - 3t^2, \qquad\qquad y(0) = \pi.$$

4. Separable Differentiable Equations:

$$\frac{dy}{dt} = f(t, y)$$

is **separable** if $f(t, y) = g(t)h(y)$ for some functions $g(t)$, $h(y)$.
Examples: $\frac{dy}{dt} = yt$ or $\frac{dy}{dt} = y^2 \sin t$ are separable, but $\frac{dy}{dt} = y - t$ is not.

5. A differential equation is **autonomous** if it is a function of the dependent variable only. (Always separable)
Example: $\frac{dy}{dt} = y^2 + e^y$.

6. Solving separable differential equations:
Example: $\frac{dy}{dt} = t^4 y$ (Note that you're really using substitution rule)

$$\frac{dy}{dt} = g(t)h(y)$$

7. Example: $\frac{dy}{dt} = y^2$
General solution consists of $y(t) = -\frac{1}{(t+c)}$ together with $y(t) = 0$.

8. Example: $\frac{dy}{dt} = ty^2 + 2y^2$, $y(0) = 1$