Existence Theorem

Suppose $f(t, y)$ is a continuous function in a rectangle of the form $\{(t, y) : a < t < b, c < y < d\}$ in the ty-plane. If (t_0, y_0) is a point in this rectangle, then there exists an $\epsilon > 0$ and a function $y(t)$ defined for $t_0 - \epsilon < t < t + \epsilon$ that solves the initial-value problem

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

(In other words, as long as the right-hand side is reasonable, solutions exist.)

Uniqueness Theorem

Suppose $f(t, y)$ and $\partial f/\partial y$ are continuous functions in a rectangle of the form

$$\{(t, y) : a < t < b, c < y < d\}$$

in the ty-plane. If (t_0, y_0) is a point in this rectangle and if $y_1(t)$ and $y_2(t)$ are two functions that solve the initial-value problem

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

for all t in the interval $t_0 - \epsilon < t < t + \epsilon$ ($\epsilon > 0$), then

$$y_1(t) = y_2(t)$$

for $t_0 - \epsilon < t < t + \epsilon$.

(In other words, the solution is unique in this interval.)