

Exam 1

Name: _____

Math 302.01
February 11, 2009

Question	Points Earned	Points Possible
1		12
2		12
3		10
4		10
5		10
6		12
7		10
Total		75

1. Determine if the statement is true or false. If it is true, explain why. If it is false, provide a counterexample or explanation.

(a) Every separable differential equation is autonomous.

(b) Every autonomous differential equation is separable.

(c) The solution of $\frac{dy}{dt} = (y + 2)(y - 3)$ with $y(0) = 0$ tends to 3 as $t \rightarrow \infty$.

(d) The function $y(t) = -e^{-t}$ is a solution to the differential equation $\frac{dy}{dt} = |y|$.

2. Give examples of each of the following:

(a) A homogeneous first-order linear differential equation.

(b) A separable differential equation that is not linear.

(c) A first-order differential equation that is autonomous, separable, linear, and homogeneous.

(d) A differential equation with equilibrium solutions at $y = 2$ and $y = 0$ and a hole at $y = -4$.

3. Solve the initial value problem $\frac{dy}{dt} = 2ty + 3te^{t^2}$ with $y(0) = 1$.

4. A 1000-gallon tank initially contains a mixture of 450 gallons of cola and 50 gallons of cherry syrup. Cola is added at the rate of 8 gallons per minute, and cherry syrup is added at the rate of 2 gallons per minute. At the same time, a well mixed solution of cherry cola is withdrawn at the rate of 5 gallons per minute. Write an **initial-value problem** to determine how many gallons of cherry syrup are present when the tank is full. (You do not need to solve the problem.) Be sure to define all variables used and give initial conditions.

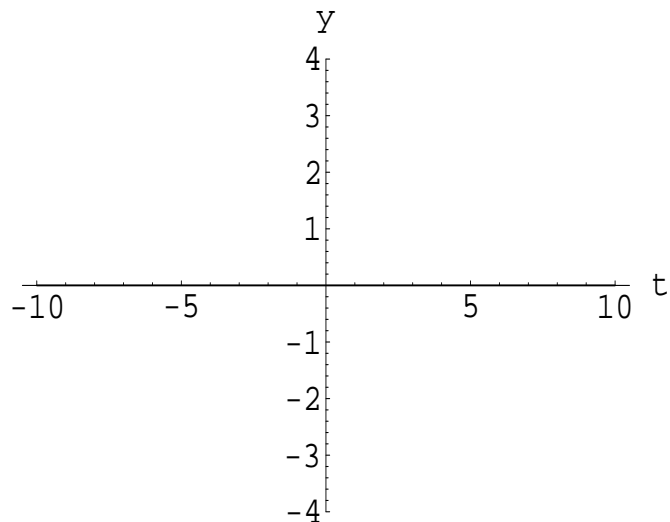
5. Find the general solution of the differential equation $\frac{dy}{dt} = \frac{ty}{1+t^2}$.

6. Consider the differential equation $\frac{dy}{dt} = y^2 - y - 2$.

(a) Find the equilibrium solutions, sketch the phase line, and classify each of the equilibrium points as a source, sink, or node.

(b) In the ty -plane, draw solution curves corresponding to each of the following initial conditions. (You will be drawing 3 curves in all.)

(i) $y(0) = -3$, (ii) $y(0) = 0$, (iii) $y(0) = 3$.



(c) What are the bifurcation value(s) of the one-parameter family $\frac{dy}{dt} = \mu y$?

7. Consider the differential equation $\frac{dy}{dt} = t^2y + 1 + y + t^2$.

(a) This is a nonhomogeneous differential equation. Find the general solution of its associated homogeneous equation.

(b) Calculate the equilibrium point of the nonhomogeneous equation.

(c) Using the Extended Linearity Principle, find the general solution of the nonhomogeneous equation.