Exam 1 Name:____________________________

Math 302.01
February 21, 2007

Question	**Points Earned**	**Points Possible**
1		12
2		12
3		10
4		10
5		10
6		12
7		10
Total		75

Math 302, Section 01 Name:________________________________

Exam 1 - February 21, 2007

1. Determine if the statement is true or false. If it is true, explain why. If it is false, provide a counterexample or explanation.

   (a) The function $y(t) = \sec t$ is a solution to the differential equation $\dfrac{dy}{dt} = y \tan t$.

   (b) Every homogeneous linear differential equation is separable.

   (c) Every separable differential equation is a homogeneous linear equation.

   (d) The solution of $\dfrac{dy}{dt} = -(y+2)(y-3)$ with $y(0) = 0$ tends to -2 as $t \to \infty$.

2. Give examples of each of the following:

   (a) A non-homogeneous first-order linear differential equation.

   (b) A first-order linear differential equation that is not separable.

   (c) A first-order differential equation that is autonomous, separable, linear, and homogeneous.

   (d) An autonomous differential equation with equilibrium solutions at $y = -3$ and $y = 0$ and a hole at $y = 1$.

3. Solve the initial value problem $\frac{dy}{dt} = 3y + 2e^{3t}$ with $y(0) = -1$.

4. Consider the differential equation $\frac{dy}{dt} = 2ty^2 + 3y^2$.

   (a) Specify if the given equation is autonomous, linear and homogeneous, linear and non-homogeneous, and/or separable.

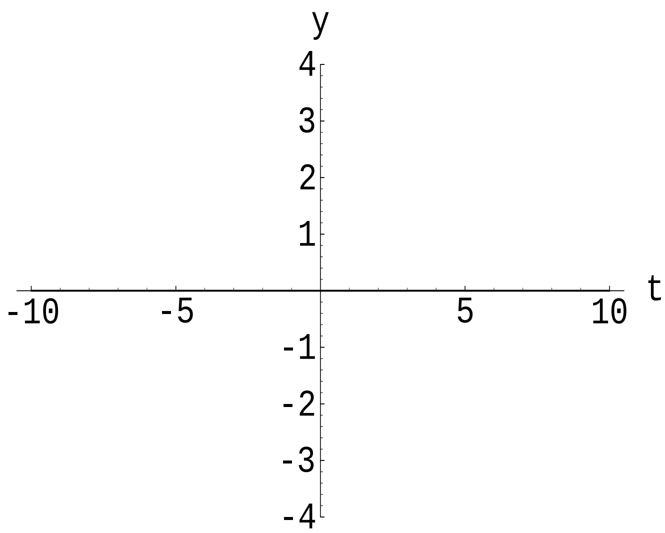
   (b) Determine the general solution to the differential equation.

5. A 30-gallon tank initially contains 15 gallons of salt water containing 6 pounds of salt. Suppose salt water containing 1 pound of salt per gallon is pumped into the top of the tank at the rate of 2 gallons per minute, while a well-mixed solution leaves the bottom of the tank at a rate of 1 gallon per minute. How much salt is in the tank when the tank is full? (You do not need to solve the problem.) Be sure to define all variables used and give initial conditions.

6. Consider the differential equation $\frac{dy}{dt} = 2y^2 + y - 3$.

(a) Find the equilibrium solutions, sketch the phase line, and classify each of the equilibrium points as a source, sink, or node.

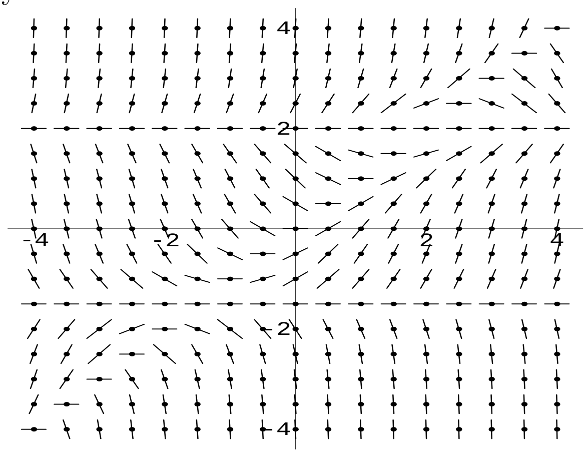
(b) In the ty-plane, draw solution curves corresponding to each of the following initial conditions. (You will be drawing 3 curves in all.)
(i) $y(0) = -3$, (ii) $y(0) = 0$, (iii) $y(0) = 3$.



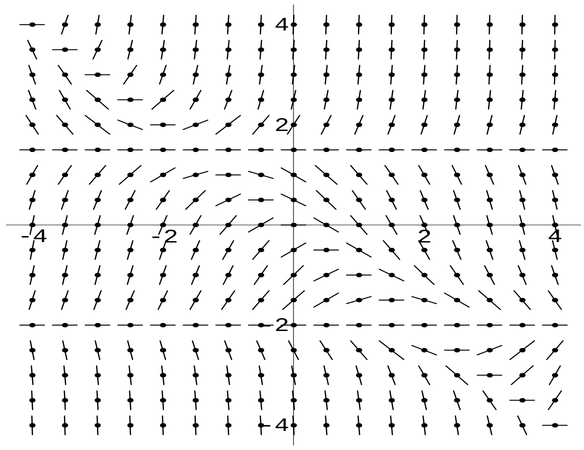
7. Which of these slope fields could represent the differential equation

$$\frac{dy}{dt} = \frac{1}{4}\left(2y+3\right)\left(y-2\right)\left(y+t\right)?$$

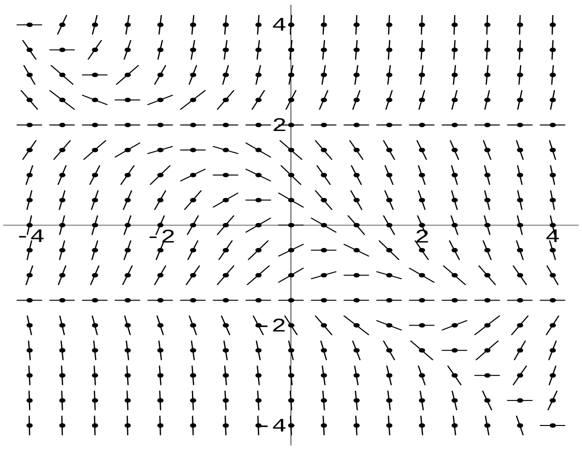
Why?



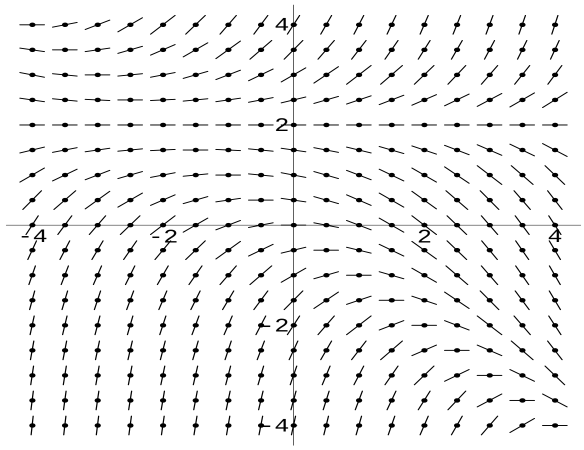
a)



b)



c)



d)