Exam 3 Name:__________________________

Math 302.01
April 25, 2007

Each problem is worth 10 points. You must complete **8** of the 9 possible problems, and you must clearly mark which **8** problems you want graded. (Circling the numbers of the problems to be graded in the table below would be a good idea.) If you do not clearly mark which problems should be graded, I will grade the first 8 problems. YOU MUST SHOW YOUR WORK/JUSTIFY YOUR ANSWER TO RECEIVE FULL CREDIT FOR A PROBLEM.

Question	**Points Earned**	**Points Possible**
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		80

Math 302, Section 01
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Name:________________________________

1. Give an **example** of a coefficient matrix $\mathbf{A}$ for a linear system $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ in which the origin would be classified as each of the following:

   (a) A saddle.

   (b) A spiral source.

2. Which of the following are eigenvectors for the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}?$$

(i) $\mathbf{v}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (ii) $\mathbf{v}_2 = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$ (iii) $\mathbf{v}_3 = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

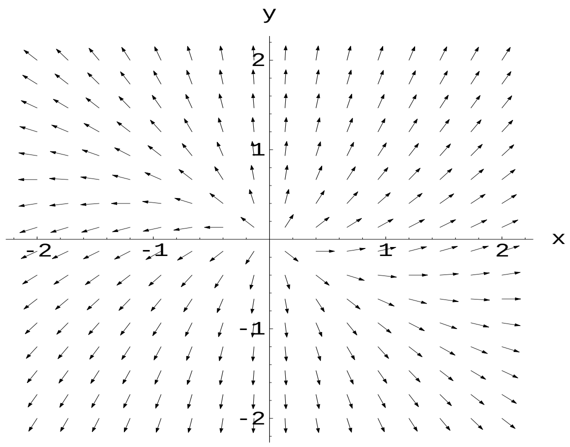
(iv) $\mathbf{v}_4 = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ (v) $\mathbf{v}_5 = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$ (vi) $\mathbf{v}_6 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

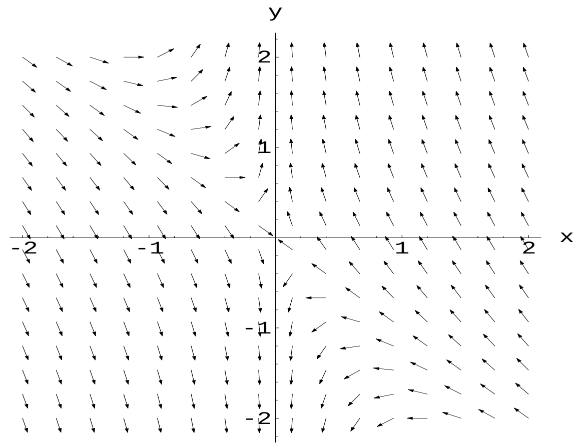
3. The linear system

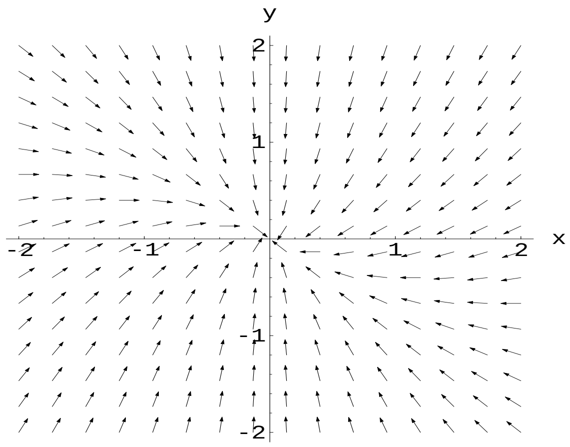
$$\frac{dx}{dt} = 3x - 4y$$
$$\frac{dy}{dt} = 4x + 3y$$

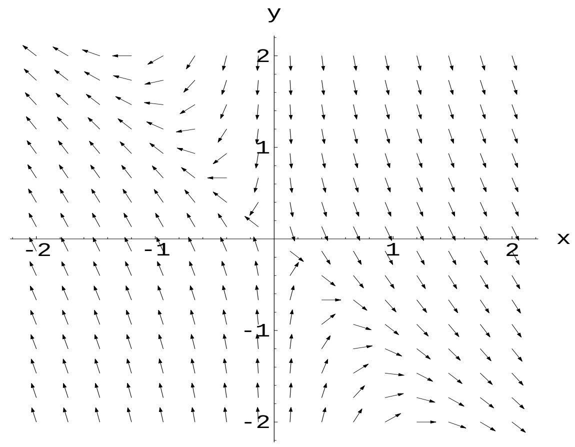
has eigenvalues $\lambda_1 = 3 + 4i$ and $\lambda_2 = 3 - 4i$ and corresponding eigenvectors $V_1 = \begin{pmatrix} i \\ 1 \end{pmatrix}$ and $V_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$. Write the general solution to this system in a form that does not involve imaginary exponents.

4. The coefficient matrix of the linear system $\dfrac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 0 \\ -5 & -3 \end{pmatrix} \mathbf{Y}$ has eigenvalues $\lambda_1 = 2$ and $\lambda_2 = -3$ with eigenvectors $\mathbf{V}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{V}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, respectively. Which of the following direction fields could correspond to this system? Why?

a) 

b) 

c) 

d) 

5. Find a particular solution to the linear system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & 1 \\ -1 & -4 \end{pmatrix} \mathbf{Y}$$

that satisfies the initial condition $\mathbf{Y}_0 = (2, 4)$.

6. Determine the general solution to the system

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = -x + 4y.$$

7. Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \mathbf{Y}.$$

Write the general solution to this system in a form that does not involve imaginary exponents.

(a) Determine a **complex solution** to this linear system.

(b) For this linear system, is the origin a spiral source, spiral sink, or center? Why?

8. Assume that a linear system has eigenvalues $\lambda_1 = 4$ and $\lambda_2 = 0$ with eigenvectors $V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $V_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, respectively.

   (a) What is the general solution for the system?

   (b) What are the equilibrium points?

9. Once again, Matt is searching for good problems to put on his differential equations exam. He wants to create a linear system whose coefficient matrix has $\lambda = 3$ as a repeated eigenvalue with eigenvector $V = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. What coefficient matrix should he use in order to have both the desired eigenvalue and eigenvector?