Exam 1

Name:____________________________

Math 302.01
September 19, 2008

Question	**Points Earned**	**Points Possible**
1		12
2		8
3		12
4		10
5		10
6		10
7		10
8		12
Total		

Math 302, Section 01 Name:________________________________

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1. Determine if the statement is true or false. If it is true, briefly explain why. If it is false, provide a counterexample or explanation.

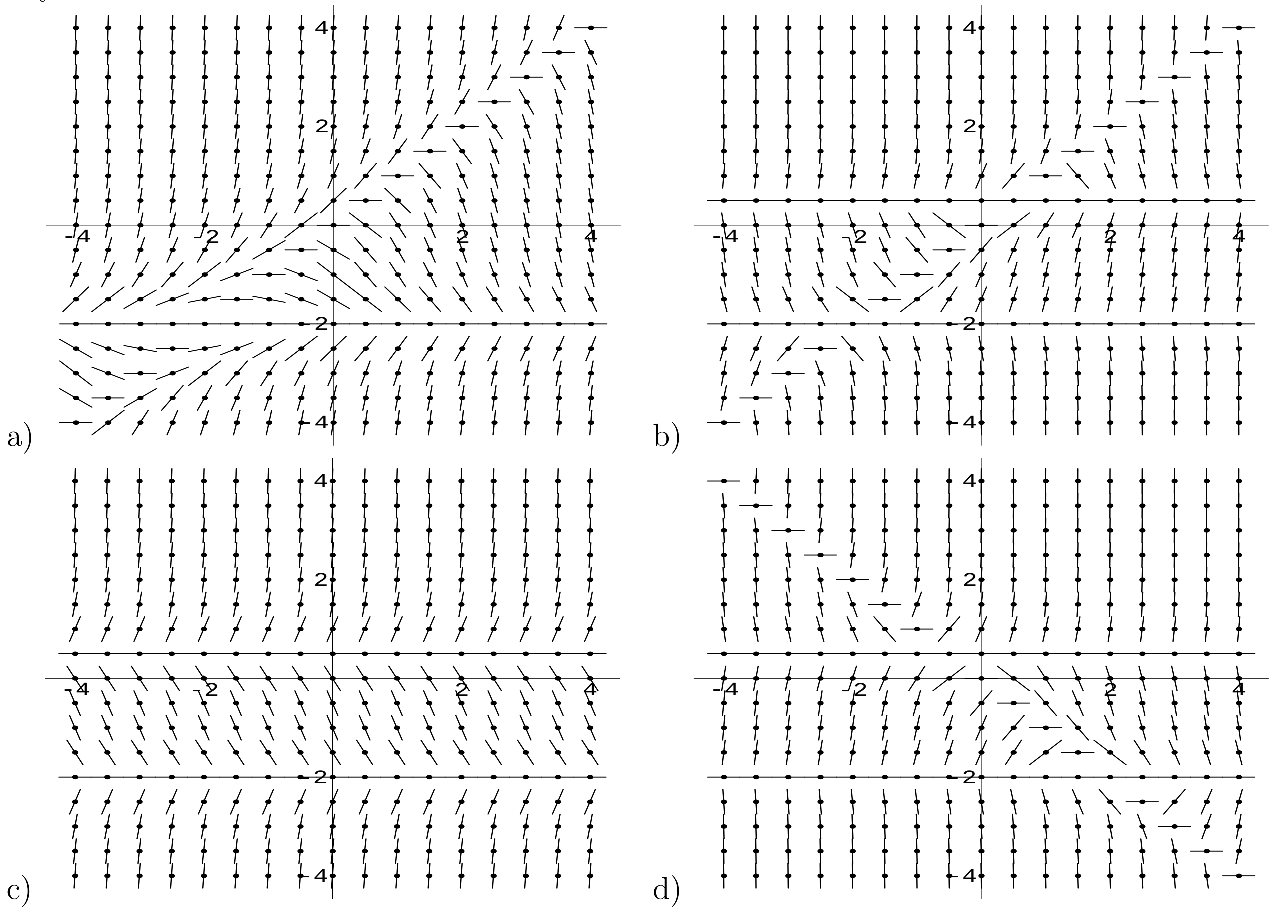
   (a) The differential equation $\frac{dy}{dt} = e^t \sin t$ produces a slope field that is parallel along vertical lines.

   (b) Suppose $f(y)$ is a continuous function for all y. The phase line for $\frac{dy}{dt} = f(y)$ must have the same number of sources as sinks.

   (c) The solution of $\dfrac{dy}{dt} = -(y+5)^2(y+2)$ with $y(0) = -3$ tends toward the equilibrium solution $y = -2$ as $t \to \infty$.

   (d) The function $y(t) = e^{t^2+3t+2}$ is a solution to the differential equation
   $$\frac{dy}{dt} = 2yt + 3y.$$

2. Which of these slope fields could represent the differential equation $\frac{dy}{dt} = (2y-1)(y+2)(y-t)$? Why?



3. Find the general solution of the differential equation $\frac{dy}{dt} = t^2 y$.

4. Give examples of each of the following:

   (a) An autonomous differential equation.

   (b) A separable differential equation that is not autonomous.

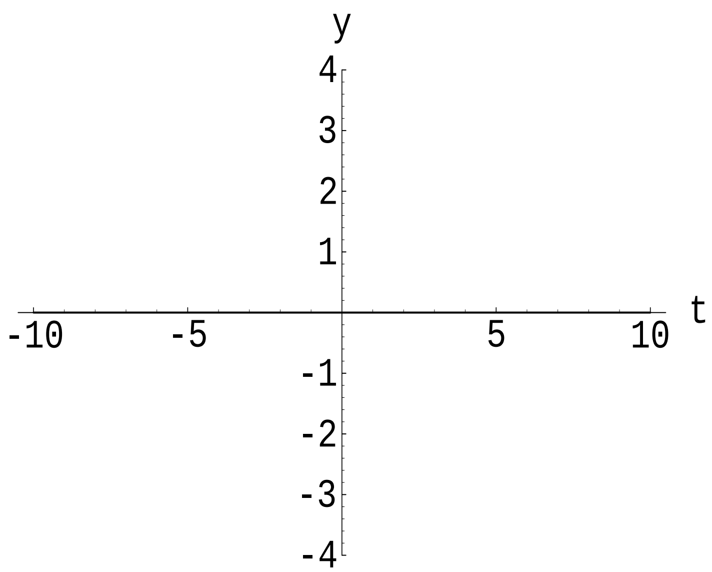
   (c) A first-order linear differential equation that is not separable.

   (d) A differential equation with equilibrium solutions at $y = 0$ and $y = -3$.

5. Consider the differential equation $\frac{dy}{dt} = (y - 2)\ln|y|$.

(a) Find the equilibrium solutions, sketch the phase line, and classify each of the equilibrium points as a source, sink, or node.

(b) In the ty-plane, draw solution curves corresponding to each of the following initial conditions. (You will be drawing 4 curves in all.)
(i) $y(0) = -2$, (ii) $y(0) = .5$, (iii) $y(0) = 1.5$, (iv) $y(0) = 3$



6. Solve the initial value problem $\frac{dy}{dt} = 3y + e^{7t}$ with $y(0) = \frac{1}{2}$.

7. The air in a small rectangular room 20 ft $\times$ 5 ft $\times$ 10ft is 3% carbon monoxide. Starting at $t = 0$, air containing 1% carbon monoxide is blown into the room at the rate of 100 ft^3 per hour and well mixed air flows out through a vent at the same rate. Write an **initial-value problem** for the amount of carbon monoxide in the room over time.
(You do not need to solve the initial-value problem.)

8. Sketch the bifurcation diagram for the differential equation

$$\frac{dy}{dt} = y^2 + \mu y + 4.$$

Include direction arrows on the phase lines and make clear the exact μ value(s) where bifurcation(s) occur(s).