

Exam 3

Name: _____

Math 302.01
November 21, 2008

Each problem is worth 10 points. You must complete **8** of the 9 possible problems, and you must clearly mark which **8** problems you want graded. (Circling the numbers of the problems to be graded in the table below would be a good idea.) If you do not clearly mark which problems should be graded, I will grade the first 8 problems. **YOU MUST SHOW YOUR WORK/JUSTIFY YOUR ANSWER TO RECEIVE FULL CREDIT FOR A PROBLEM.**

Question	Points Earned	Points Possible
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		80

1. Give an **example** of a coefficient matrix **A** for a linear system $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ in which the origin would be classified as each of the following:

(a) A source. (**Not** a **spiral** source)

(b) A spiral sink.

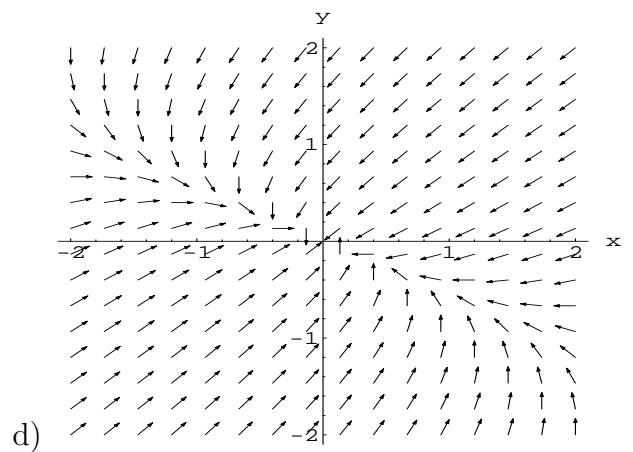
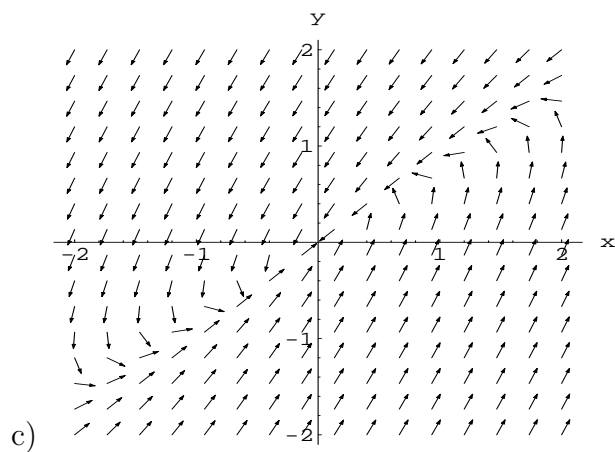
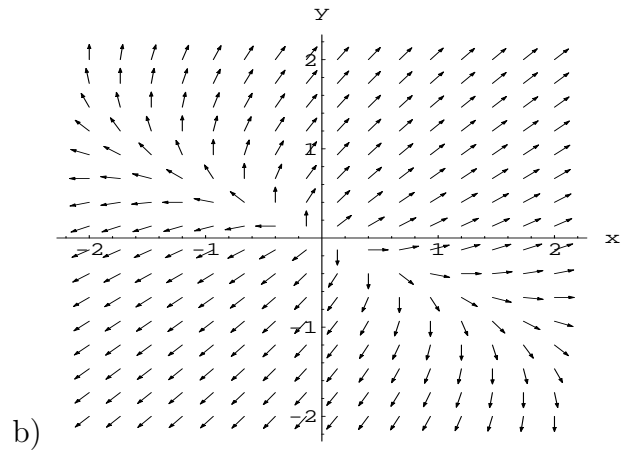
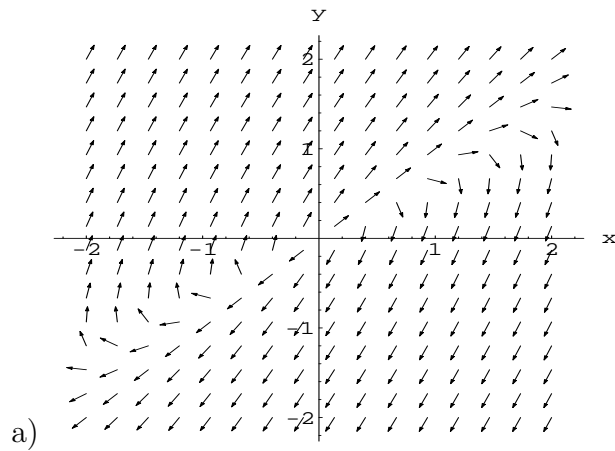
2. Assume that a matrix \mathbf{A} has eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$ with eigenvectors $\mathbf{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{V}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, respectively. Determine a solution to the initial value problem

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}, \quad \mathbf{Y}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

3. Determine the solution to the initial-value problem

$$\begin{aligned} \frac{dx}{dt} &= -3x - y \\ \frac{dy}{dt} &= x - 5y, \quad \mathbf{Y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \end{aligned}$$

4. The coefficient matrix of the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & -2 \\ -1 & -3 \end{pmatrix} \mathbf{Y}$ has eigenvalues $\lambda_1 = -4$ and $\lambda_2 = -1$ with eigenvectors $\mathbf{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\mathbf{V}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, respectively. Which of the following direction fields could correspond to this system? Why?



5. The linear system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \mathbf{Y}$$

has eigenvalues $\lambda_1 = 1 + 2i$ and $\lambda_2 = 1 - 2i$ and corresponding eigenvectors $V_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$ and $V_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$. Write the general solution to this system in a form that does not involve imaginary exponents.

6. Consider the linear system of differential equations

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = ax + 4y.$$

(a) For which values of a will the system have two distinct, real eigenvalues?

(b) For what value of a will the system have one repeated, real eigenvalue?

(c) For which values of a will the system have two complex eigenvalues?

7. Find the solution to the initial-value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$

8. Find the general solution to the linear system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix} \mathbf{Y}.$$

9. As always, Matt is struggling to come up with good problems for a differential equations exam. He wants to create a linear system in which every point on the line $y = x$ is an equilibrium point. He knows that the only way a linear system will have equilibrium points other than the origin is if one row of the coefficient matrix is a multiple of the other. (Or equivalently, the determinant of the coefficient matrix is zero.) Give an example of a coefficient matrix for which every point on the line $y = x$ is an equilibrium point.