Section 1.1 - Modeling via Differential Equations

1. Approaches to the study of the solutions of differential equations:
   - Analytic - Searches for explicit formulas that describe the behavior of solutions.
   - Qualitative - Uses geometry to give an overview of the behavior of the model (Does not give precise values of the solution at specific times)
   - Numerical - A computer approximates the solutions we seek.
2. Model: Gives a representation of some aspect of an object.
   Note: The "best" model depends on how we use the model.
3. Basic steps for creating a model: (p. 3)
   - **Step 1:** Clearly state the assumptions on which the model will be based. These assumptions should describe the relationships among the quantities to be studied.
   - **Step 2:** Completely describe the variables and parameters to be used in the model – "you can't tell the players without a program."
   - **Step 3:** Use the assumptions formulated in Step 1 to derive equations relating the quantities in Step 2.
4. Model Quantities:
   (a) Independent Variable - independent of any other quantity (almost always time)
   (b) Dependent Variables - quantities that are functions of the independent variable
   (c) Parameters - quantities that don't change with time but can be adjusted
5. Example: $\dfrac{dP}{dt} = kP$
   - First-order: contains only first derivatives
   - Ordinary differential equation: contains no partial derivatives
   - Equilibrium solution: constant for all values of the independent variable
6. Example: (Problem 2, p. 14)
   Consider the population model
   $$\frac{dP}{dt} = 0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P,$$
   where $P(t)$ is the population at time t.
   (a) For what values of P is the population in equilibrium?
   (b) For what values of P is the population increasing?
   (c) For what values of P is the population decreasing?

7. Example: $\dfrac{dP}{dt} = kP$ for all t and $P(0) = P_0$

   - Initial Condition
   - Initial-Value Problem
   - Solution (to an initial-value problem)
   - Particular Solution
   - General Solution

8. Logistic Population Model with Growth Rate k and Carrying Capacity N.

$$\frac{dP}{dt} = k\left(1 - \frac{P}{N}\right)P$$