

Section 1.3 - Qualitative Technique: Slope Fields

1. Slope Fields:

Example: $\frac{dy}{dt} = 2y - t$;

Sketch by hand; then, use HPGSolver. (Also draw in certain curves)

2. Slope fields of

- $\frac{dy}{dt} = f(t)$: Slope marks on each vertical line are parallel. Ex. $\frac{dy}{dt} = t^2 + 1$
Can get infinitely many solutions from one solution. (Translate up or down)
- $\frac{dy}{dt} = f(y)$: Slope marks on each horizontal line are parallel. Ex. $\frac{dy}{dt} = 4y^2$ (Autonomous)
(Observe equilibrium solutions)
Can get infinitely many solutions from one solution. (Translate left or right)

3. Qualitative methods don't answer specific questions, but they can help us understand long-term behavior. Also, some autonomous differential equations involve evaluating difficult integrals but can be looked at quantitatively

(Example: $\frac{dy}{dt} = e^{y^2} \sin 3y$).

4. Example: An RC Circuit (p. 44)

- Resistance: R (positive parameter)
- Capacitance (behavior of the capacitor): C (positive parameter)
- Input voltage across voltage source: $V(t)$
- Current: $i(t)$
- Voltage across capacitor: $v_c(t)$

$v_c(t)$ satisfies $RC \frac{dv_c}{dt} + v_c = V(t)$, which is equivalent to $\frac{dv_c}{dt} = \frac{V(t) - v_c}{RC}$.

Zero input:

If $V(t) = 0$, then the equation becomes $\frac{dv_c}{dt} = \frac{-v_c}{RC}$. Pick $R = 0.3$ and $C = 2$ and look at the slope field. (All solutions decay toward $v_c = 0$ as t increases)

Constant Nonzero Voltage Source:

If $V(t) = K \neq 0$, then

$$\frac{dv_c}{dt} = \frac{K - v_c}{RC}.$$

(Autonomous with one equilibrium solution as $v_c = K$.)

(All solutions tend toward equilibrium as t increases.)

Given any initial voltage $v_c(0)$ across the capacitor, the voltage $v_c(t)$ tends to $v = K$ as t increases.

Example: Look at $\frac{dv_c}{dt} = \frac{K - v_c}{RC}$ where $R = 0.3$ and $C = 2$ and vary K ($K = 0, 1, 5$).

Note: We could find a formula for the general solution by separating variables and integrating.