

## Section 1.3 - Qualitative Technique: Slope Fields

### 1. Slope Fields:

Example:  $\frac{dy}{dt} = 2y - t$ ;

Sketch by hand; then, use HPGSolver. (Also draw in certain curves)

### 2. Slope fields of

- $\frac{dy}{dt} = f(t)$ : Slope marks on each vertical line are parallel. Ex.  $\frac{dy}{dt} = t^2 + 1$   
Can get infinitely many solutions from one solution. (Translate up or down)
- $\frac{dy}{dt} = f(y)$ : Slope marks on each horizontal line are parallel. Ex.  $\frac{dy}{dt} = 4y^2$  (Autonomous)  
(Observe equilibrium solutions)  
Can get infinitely many solutions from one solution. (Translate left or right)

### 3. Qualitative methods don't answer specific questions, but they can help us understand long-term behavior. Also, some autonomous differential equations involve evaluating difficult integrals but can be looked at quantitatively

(Example:  $\frac{dy}{dt} = e^{y^2} \sin 3y$ ).

### 4. Example: An RC Circuit (p. 44)

- Resistance:  $R$  (positive parameter)
- Capacitance (behavior of the capacitor):  $C$  (positive parameter)
- Input voltage across voltage source:  $V(t)$
- Current:  $i(t)$
- Voltage across capacitor:  $v_c(t)$

$v_c(t)$  satisfies  $RC \frac{dv_c}{dt} + v_c = V(t)$ , which is equivalent to  $\frac{dv_c}{dt} = \frac{V(t) - v_c}{RC}$ .

#### Zero input:

If  $V(t) = 0$ , then the equation becomes  $\frac{dv_c}{dt} = \frac{-v_c}{RC}$ . Pick  $R = 0.3$  and  $C = 2$  and look at the slope field. (All solutions decay toward  $v_c = 0$  as  $t$  increases)

#### Constant Nonzero Voltage Source:

If  $V(t) = K \neq 0$ , then

$$\frac{dv_c}{dt} = \frac{K - v_c}{RC}.$$

(Autonomous with one equilibrium solution as  $v_c = K$ .)

(All solutions tend toward equilibrium as  $t$  increases.)

Given any initial voltage  $v_c(0)$  across the capacitor, the voltage  $v_c(t)$  tends to  $v = K$  as  $t$  increases.

Example: Look at  $\frac{dv_c}{dt} = \frac{K - v_c}{RC}$  where  $R = 0.3$  and  $C = 2$  and vary  $K$  ( $K = 0, 1, 5$ ).

Note: We could find a formula for the general solution by separating variables and integrating.