Section 2.1 - Modeling via Systems

1. The Predator-Prey System (Revisited)
   $R(t)$ - prey population; $F(t)$ - predator population

$$\frac{dR}{dt} = 1.8R - 1.3RF$$

$$\frac{dF}{dt} = -F + 0.8RF$$

- Look at $R = 0$, $F = 0$: absence of prey/predators
- Equilibrium Solutions:
  (a) $R(t) = 0$, $F(t) = 0$ (Trivial-makes sense)
  (b) $R(t) = 10/8 = 1.25$, $F(t) = 1.8/1.3 \approx 1.3846$ (System in balance)
- If $R(t) = 0$, then we have $\frac{dF}{dt} = -F$ (Exponential decay)
- If $F(t) = 0$, then we have $\frac{dR}{dt} = 1.8R$ (Exponential growth-logistic might be better)
- $R(t)$ and $F(t)$ graphs:
  - Initial condition now (R_0, F_0)
  - Initial condition yields a solution $(R(t), F(t))$ that satisfies the system.
  - Bad news: Few analytic techniques; Good news: Numerical/Qualitative methods
  - Use HPGSystemSolver to look at graphs of $F(t)$, $R(t)$ with $(F_0, R_0) = (1, 0.5)$
  - Observe periodic
- Phase Portrait: (Focus on $R \geq 0$, $F \geq 0$)
  - Phase Plane: RF-plane
  - Solution curve: As t varies, $(R(t), F(t))$ sweeps out a curve in the RF-plane
  - Initial point (1, 0.5) - look at the direction in which it moves (counter-clockwise)
  - Phase portrait: many solutions plotted simultateously
  - Equilibrium points: plotted as points (Observe locations)

2. Modified Predator-Prey Model:

$$\frac{dR}{dt} = 2R\left(1 - \frac{R}{2}\right) - 1.2RF$$

$$\frac{dF}{dt} = -F + 0.9RF$$

   When $F = 0$, prey population obeys logistic growth model with carrying capacity 2.

- Equilibrium solutions at $(0, 0)$, $(2, 0)$, and $(10/9, 20/27) \approx (1.11, 0.74)$.
- Look at in HPGSystemSolver

3. The Motion of a Mass Attached to a Spring

   (a) Newton's 2^{nd} Law: $F = ma$

   (b) If $y(t)$ is position of mass at time t, then $F = m\frac{d^2y}{dt^2}$

   (c) Restoring force by a spring: $F_s = -ky$ (Spring constant: $k > 0$)

   (d) m and k are parameters

   (e) Combining these, we get

$$F_s = -ky = m\frac{d^2y}{dt^2}, \text{ or}$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

   Second equation is called a simple harmonic oscillator (second-order differential equation)

   (f) Recall that $v = \frac{dy}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2y}{dt^2}$.

   (g) It follows that we can rewrite the second-order differential equation as:

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = -\frac{k}{m}y$$