Section 2.5 - The Lorenz Equations

1. Progression from Phase Line to Phase Plane
   - Solutions have more room to move in a plane than on a line.
   - Uniqueness Theorem still provides restrictions on possible phase plane pictures. Solution curves in the phase plane don't cross.
2. Consider raising number of dependent variables to 3 (still autonomous):
   - 3-dimensional phase space: Uniqueness Theorem still applies-solutions can't cross.
   - Curves can knot an link in complicated ways (Knot Theory)
   - Computers can approximate complicated solution curves.
   - Problem: make sense of the pictures.
3. Lorenz Equations - first studied by Ed Lorenz (1963) to model the weather

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = \rho x - y - xz$$
$$\frac{dz}{dt} = -\beta z + xy$$

   x, y, and z are dependent variables; α, β, and ρ are parameters.

   System is much simpler than the one for modeling the weather, but by studying it, Lorenz started a scientific revolution by making scientists and engineers aware of chaos theory.

4. Example: Lorenz chose to study the system with $\sigma = 10$, $\beta = 8/3$, and $\rho = 28$.

$$\frac{dx}{dt} = 10(y - x)$$
$$\frac{dy}{dt} = 28x - y - xz$$
$$\frac{dz}{dt} = -\frac{8}{3}z + xy$$

   The equilibrium points are $(0, 0, 0)$, $(6\sqrt{2}, 6\sqrt{2}, 27)$, and $(-6\sqrt{2}, -6\sqrt{2}, 27)$. (Exercise 1)
   Look at examples with $(x_0, y_0, z_0) = (0, 1, 0)$ and $(x_0, y_0, z_0) = (0, 1.001, 0)$.
   See textbook for diagram, and see excel for spreadsheet.

5. Important Properties of the Lorenz system
   - A small change in initial conditions leads fairly quickly to large differences in corresponding solutions.
   - Although the details of individual solutions are quite different, the pictures of the solution curves in three-dimensional space look remarkably alike.