

Section 13: Euler's Formula, Polytopes

1. Polygons (Convex)
2. Definitions:
 - Faces of a graph
 - Infinite face
3. Polyhedra (3-dimensional, convex)
 - f -vector (V, E, F) : Give examples and non-examples
 - Euler's relation $V - E + F = 2$ (Proof by eraser)
 - $2E \geq 3F$
 - $2E \geq 3V$
 - Steinitz' Theorem: $F \leq 2V - 4$, or $F \geq \frac{1}{2}V + 2$
 - $6 \leq 3F - E \Rightarrow 3F_3 + 2F_4 + F_5 + 0F_6 - F_7 - 2F_8 - 3F_9 - \dots$ (p. 69, pr. 13.6)
 - All triangles $\Rightarrow F_3 \geq 4$
 - Quadrilaterals and Hexagons $\Rightarrow F_5 \geq 12$
 - Pentagons and Hexagons $\Rightarrow F_5 \geq 12$
 - Other things to prove:
 - Simplicial \Rightarrow Even # of faces.
 - Can't have exactly 7 edges.